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Analysis of a Bucket Brigade for Order Picking

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Abstract

Bucket brigade order picking is a method for retrieving orders from a storage rack where workers follow a fixed sequence and dynamically adjust to variability in work content along the rack. The method is simple and has been shown to provide superior performance in many applications. We develop an analytic model for predicting performance as a function of design and environmental parameters, and we use properties of the model in conjunction with simulation to offer insights into bucket brigade productivity.

Keywords: warehouse management, order picking, bucket brigade

Analysis of a Bucket Brigade for Order Picking

1. Introduction

The amount of on-line shopping has been steadily increasing in recent years (e.g., Holzwarth, Janiszewski, and Neuman 2006 report that internet retail sales grew at an average annual rate of 30% from 2001-2005). However, one area that has limited growth has been the performance of order fulfillment centers (Boyer and Hult 2006). Where delivery speed was once thought of as method of differentiation, it is quickly becoming an order qualifier as customers are demanding the on-line shopping experience resemble that of traditional retail to the greatest extent possible. Customers now expect accurate orders to be processed and on their way within a few hours of the time they are placed. The ability to meet this requirement efficiently is a key competitive priority in this industry.

While fulfillment centers perform a variety of functions, the core activity is the retrieval of customer orders from storage. This function is termed *order picking* and accounts for more than half of the costs and labor requirements in a typical fulfillment center (Loudin 2001). The order picking function in e-commerce applications is particularly complex since small orders comprised of one-to-three items are the norm (Cooke 2000).

A common approach to order picking in the e-commerce environment is to partition the warehouse into multiple modules (cells) each consisting of a line of gravity flow racks (i.e., racks tilted forward and loaded from the back). Each cell contains the same population of stock keeping units (SKUs) allowing for the flexibility of retrieving orders from any of the cells. The cells are staffed by a team of order pickers assigned to fixed zones along the line. Orders are picked into totes that proceed along the line on a passive conveyer from zone to zone until the order is complete. In this configuration (referred to in the industry as 'zone picking' or 'pick and pass'), the throughput of the system is largely dictated by the ability to determine zone boundaries that balance the work among order pickers. This is a difficult task given the variability in content within and between orders and the difference in worker speeds that is common in the industry.

An alternative approach to order picking in this environment termed *bucket brigades* has been proposed recently. A bucket brigade is a decentralized method for organizing sequential tasks performed by a group of workers and uses no fixed zones. It operates according to the following simple rule: each worker moves forward completing tasks (i.e., retrieving items) in sequence until the work is complete or another worker takes over, at which point the worker reverses direction to take over work from the upstream worker, or in the case of the worker in the first position, begins work on the first task of a new job. This simple mechanism allows workers to adjust dynamically to variability in work content. This property is not shared by a zone system and, consequently, bucket brigades are generally considered the preferred alternative for order retrieval in fulfillment centers (Bartholdi, Eisenstein, and Foley 2001).

A potential weakness of bucket brigades is susceptibility to blocking. Workers maintain their relative positions (i.e., do not pass one another) in a bucket brigade. Consequently, blocking occurs when the upstream worker is waiting behind his adjacent downstream worker who is in the midst of picking an item from the rack. Reducing blocking in a bucket brigade increases output, (optimal output can only attained by eliminating blocking), and provides advantages related to the stability in the work performed by each worker. Ordering workers from slowest-to-fastest in a bucket brigade is one way to reduce blocking (Bartholdi and Eisenstein 1996, Bartholdi, Bunimovich, and Eisenstein 1999, Bartholdi, Eisenstein, and Foley 2001). Order batching is another alternative that has been suggested (Bartholdi and Eisenstein 2006). In addition to reducing the incidence of blocking, increasing the *batch size* increases the production rate (Bartholdi and Eisenstein 1999). In a fulfillment center, time spent walking can be a significant factor in picking rates because there can be large distances between the locations of two consecutive items on a pick list. Increasing the batch size decreases the ratio of walking to picking time and, hence, increases the picks per period.

In summary, two guiding principles for the management of bucket brigades have been established in the literature to date: (1) order workers from slowest-to-fastest, and (2) increase the batch size to the point where either the number of items in a batch becomes difficult to move or the batches in process approach available space constraints.

In this paper we examine these principles and the performance of bucket brigades in general in conjunction with two key strategies for improving productivity that have been suggested in the warehousing literature.

First, we consider the possibility of adding a 'take-away' conveyor parallel to the storage rack. This option reduces walking time by eliminating the need to carry completed batches to the end of the storage rack. Barthold, Eisenstein, and Foley (2001) present an illustration of this option (p. 714) and describe its use at Revco drugs. However, a detailed analysis of the impact of adding such a conveyor to the standard configuration is not available in the current literature. We perform this analysis and find the impact of adding a take-away conveyor to be substantial both in absolute terms and with respect to the effect of other factors considered in this research. The two options (labeled C1 and C2) are depicted in Figure 1.

Second, we examine the impact of manipulating the distribution of SKUs across the storage rack. We refer to this distribution as the *storage bay activity profile.* Pick lists, which contain the items to be retrieved in each batch, are printed at one end of the storage rack, say the left end. The items in a pick list are sorted by location in the storage rack, from left to right. Assigning items to storage slots randomly results in a uniform storage bay activity profile. Management may also decide to distribute the SKUs along the rack based on volume (e.g., fast movers in one section and slow movers in another). This option has been studied extensively in rack and aisle systems (e.g., see Jarvis and McDowell 1991 and Ruben and Jacobs 1999) but has not been treated in the bucket brigade literature to date.

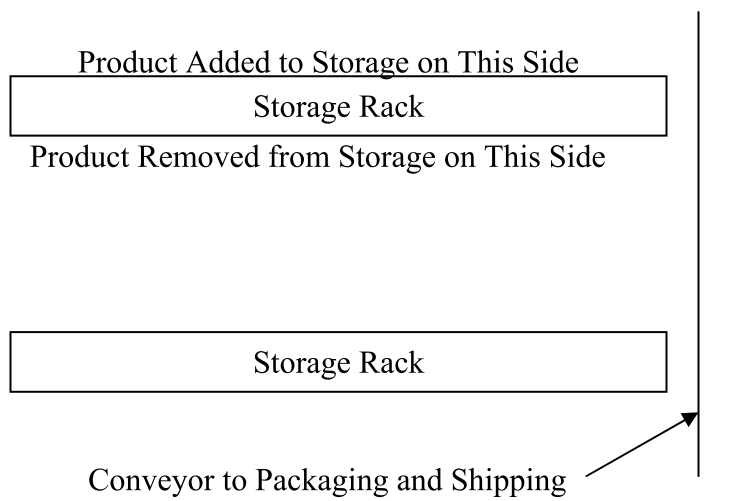

Illustration of C1 - conveyor running perpendicular to the storage rack.

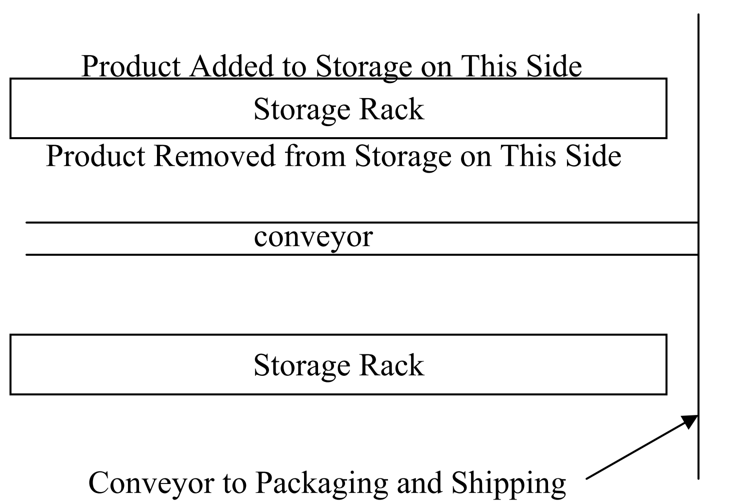

Illustration of C2 - conveyor running parallel to the storage rack that feeds a conveyor running perpendicular to the storage rack.

Figure 1: Conveyor configurations considered in this research

For a given workforce, management's decisions on conveyor configuration, batch size, storage bay activity profile, and the ordering of workers along the storage rack influences the output rate through the effect of these decisions on three key factors: (1) the degree to which the picking and walking speeds of the workers match the work content in their typical work area, (2) the degree to which average walking work content is reduced, and (3) the incidence of blocking.

We take a two-stage approach to the problem. First, we introduce a deterministic model to determine the expected allocation of work along a storage rack, and consequently, the output rate as a function of the factors discussed above. Our model allows us to determine the output rate for any system that attains long term balance and, as a result, provides insight into the potential effects of the factors we examine. We then introduce variability into the system via a detailed simulation model. Comparing the output rates from the deterministic model to those from the simulation allows us to assess the robustness of the predictions of the deterministic model. As a result, we are able to offer a variety of insights into the performance of bucket brigades in order fulfillment centers.

The next section outlines the related literature. Section 3 introduces the deterministic model and its properties. Section 4 reports the results from our simulation experiments, and Section 5 summarizes the managerial implications from our analysis.

2. Related Literature

Bartholdi and Eisenstein (1996) define and analyze the first formal model of a bucket brigade system. The model, which is motivated by a serial production line, assumes deterministic work requirements at each work station and instantaneous walk times. Bartholdi, Bunimovich, and Eisenstein (1999) analyze the asymptotic behavior of this model for the cases of two and three workers.

Bartholdi, Eisenstein, and Foley (2001) extend the Bartholdi and Eisenstein (1996) deterministic model by introducing random processing times. They assume each workstation requires work with the task time for each worker at each work station drawn from an exponential distribution. In the context of order picking in a warehouse, a work station in a production line is analogous to a particular section of a storage rack. They note that the stochastic model provides a closer match for order picking in a warehouse. The authors show that the dynamics of the stochastic model are similar to the deterministic model as long as there are a sufficient number of work stations.

The fulfillment centers which are the focus of this paper are not as well represented as a sequence of exponential processes. In particular, each batch will require picking work at only a small subset of work stations. As a result, for each batch, the work at the majority of workstations is the effort to walk by them and the time taken to complete this work will tend to be relatively constant. All models available to date assume that the forward work velocity of each worker can be represented by a single parameter and, as a result, are unable to capture this effect. The model we present below allows us to express each worker's velocity as a function of their walking and picking speed.

Bartholdi and Eisenstein (2005) extend their earlier deterministic model (Bartholdi and Eisenstein 1996) to account for walk-back times and the time required for a hand-off between two workers. The motivating application comes from a production environment that, relative to the literature, is distinct with respect to the significance of walk-back and hand-off times. The authors show that the work content of each worker stabilizes to a fixed set of tasks, which enhances learning. This is a key effect that led to a higher production rates when a firm switched from a craft assembly system with no walk-back or hand-off times to a bucket brigade.

The above models assume that relative worker speeds are constant over the production line or storage rack. Armbruster and Gel (2006) study a model of a bucket brigade production line with two workers. One worker is faster over one part of the line and slower over the other part of the line. They investigate the dynamics of the system to identify conditions under which the work content of each worker stabilizes, and how the output rate is affected by various implementation decisions.

The model in this paper uses concepts from the deterministic model of Bartholdi and Eisenstein (1996), but builds on this framework to capture the interaction between the storage bay activity profile, the distribution of work content, and the role of labor specialization. It shares a similar spirit with Armbruster and Gel (2006) in the sense that the relative speeds of workers are not constant, but it differs in the sense that relative work speeds depend on the storage bay activity profile. In general, the storage bay activity profile not only affects how the nature of work varies with location (e.g., ratio of picking-to-walking activity), but when the conveyor is along the rack, it also affects the total work content. For example, a storage bay activity profile that concentrates high volume items at the beginning of the storage rack minimizes the average distance required to pick all of the items in a batch. In addition, when picking-to-walking efficiencies vary across the workforce, the ordering of workers influences the degree to which the effects of labor specialization affect the output rate.

3. Model and Analysis

In this section, we develop the deterministic model. We separate the order picking effort into its two components (walking and picking) and assume that pick times and walk times are constant and deterministic. A pick list is always available when requested by a worker (i.e., the system is never starved for work). The pick locations of items in a batch are random and dictated by the storage bay activity profile. We then transform this stochastic model into a deterministic model based on expected values.

This section is divided into three subsections. Subsection 3.1 introduces the stochastic model. Subsection 3.2 introduces a parsimonious model for representing a wide variety of storage bay activity profiles. Subsection 3.3 describes the transformation of the stochastic model to the deterministic model, and presents a series of properties relating the output rate and the optimal design choices.

3.1. Stochastic Model of a Bucket Brigade Order Picking System

Our stochastic model relies on two key assumptions. First, we assume that there are two workers.[1] This assumption allows us to develop closed form expressions for the output rate and, as noted above, others have used this assumption (Bartholdi, Bunimovich, and Eisenstein 1999, Armbruster and Gel 2006). Bucket brigades with two or three workers are common in apparel warehousing (Bartholdi, Bunimovich, and Eisenstein 1999). Second, we assume that the batch size, which is the number of items in a pick list, is constant. This assumption is uncontroversial. Ruben and Jacobs (1999) demonstrate, in a similar environment, a simple bin-packing heuristic which retains the iid nature of the locations of items in each batch while ensuring more than 99.9% of batches consist of the same number of items.

The stochastic model assumptions and notation are listed below.

Assumptions

A1. There are two workers with worker 1 in the first position and worker 2 in the second position

A2. Batch size is constant

A3. Pick times and walk times are deterministic

[1] The analytical properties extend to case of more than two workers if the workers can be divided into two classes where workers in each class pick and walk at the same rates and each class has the same number of workers (Webster, Ruben, and Yang 2006).

A4. The rack is oriented as in Figure 1 with pick lists available at the left edge of the rack; workers pick from left-to-right; the unit of distance is selected so that the length of rack is 1

A5. The locations of items in a pick list are random and governed by a continuous probability distribution

A6. The Pareto curve for the products conforms to a power distribution (this assumption is explained the next subsection)

Notation

q = batch size, i.e., number of items in each pick list

p_i = pick time per item for worker $i \in \{1, 2\}$, which is deterministic

$\bar{p} = (p_1 + p_2)/2$ = standard pick time per item

$\rho = p_1/\bar{p} - 1$ = percentage increase in worker 1 pick time relative to standard (and $-\rho$ = percentage decrease in worker 2 pick time relative to standard) $\in (-1, 1)$

w_i = time it takes for worker $i \in \{1, 2\}$ to walk twice the length of the storage rack (i.e., to the end of the rack and back), which is deterministic

$\bar{w} = (w_1 + w_2)/2$ = standard walk time

$\omega = w_1/\bar{w} - 1$ = percentage increase in worker 1 walk time relative to standard (and $-\omega$ = percentage decrease in worker 2 walk time relative to standard) $\in (-1, 1)$

$F(y)$ = probability that an item in a pick list is located no more than y distance units from the left edge of the rack; $F(y)$ is a continuous cumulative probability distribution defined on [0, 1], and the corresponding density function is denoted $f(y)$; the function $F(y)$, or equivalently $f(y)$, defines the storage bay activity profile for the rack

Y_j = location of an item in a pick list; $Y_1, \ldots, Y_q$ are iid random variables with distribution $F(y)$

$Y_{(q)} = \max\{Y_1, \ldots, Y_q\}$ = location of the last item in a batch (i.e., $Y_{(q)}$ is the q^{th} order statistic of q iid random variables)

3.2. Pareto Curve of the Product Line as a Power Distribution

Management can select from multiple storage bay activity profiles, but the range of choice is influenced by the degree of volume variation in the product line. In this subsection we describe a measure of volume variation.

Suppose that the firm's product line is sorted from highest-to-lowest volume with $F_{-1}(y)$ denoting the corresponding cumulative distribution function; $f_{-1}(y)$ is the corresponding density function. The function $F_{-1}(y)$ is the *Pareto curve* of the product line. For example, $F_{-1}(0.2) = 0.8$ means that the top 20% of the product line makes up 80% of total volume.

We assume that the Pareto curve of the product line is a power distribution, i.e.,

$$f_{-1}(y) = \frac{1}{\eta} y^{\frac{1}{\eta}-1} \text{ and } F_{-1}(y) = y^{\frac{1}{\eta}} \tag{1}$$

where $\eta \geq 1$. The power distribution is a model of a Pareto curve with a single parameter η that reflects the degree of volume variation in a firm's product line (e.g., see Hausman, Schwarz, and Graves 1976). The larger the value of η, the more stratified the product line with respect to volume. If $\eta = 1$, for example, then the volumes of all products are identical.

If a firm's entire product line conforms to the power distribution, then a partitioning of the product line by volume (e.g., groups of SKUs to be stored in different racks) results in groups of products that also conform to the power distribution (e.g., $F_{-1}(y \mid y \leq \alpha) = (y/\alpha)^{1/\eta}$). Consequently, we let $F_{-1}(y)$ denote the Pareto curve for the portion of the product line to be placed in the storage rack. The following figure illustrates $F_{-1}(y)$ for various values of η. At η = 8, for example, the top 20% of the product line contributes 81.8% of total volume.[2]

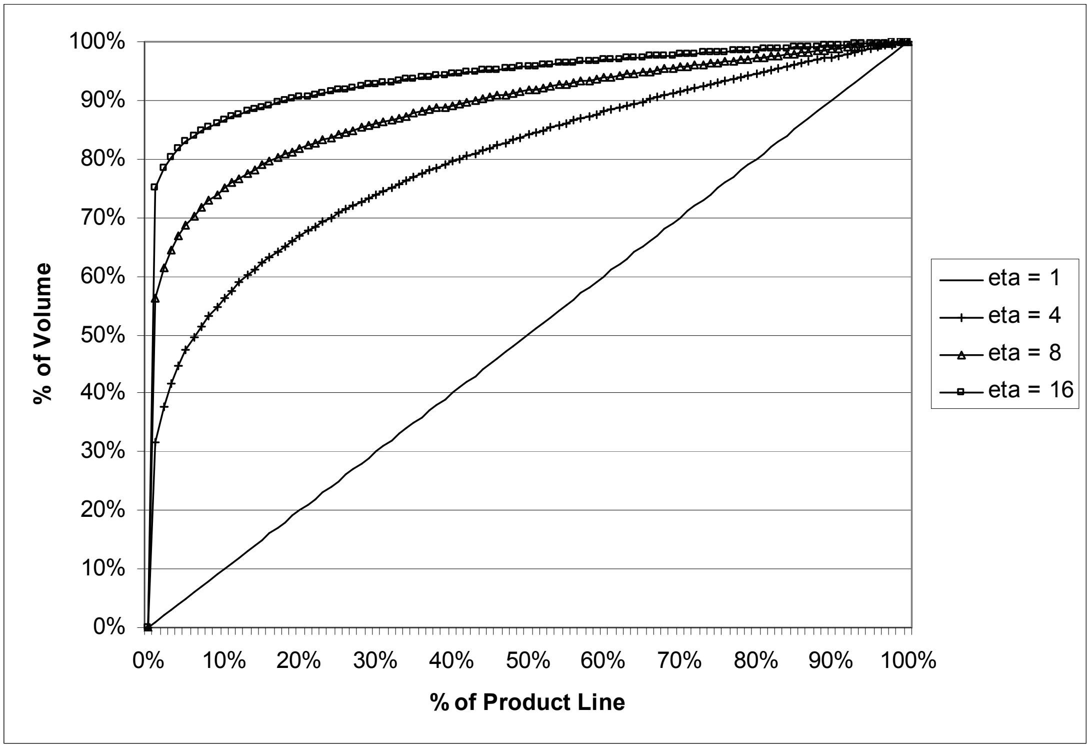

Figure 2. Pareto curve of the product line as a power distribution, $F_{-1}(y) = y^{1/\eta}$, for various values of η.

Reflecting $f_{-1}(y)$ about the vertical axis at $y = 0.5$, we get

$$f_1(y) = \frac{1}{\eta}(1-y)^{\frac{1}{\eta}-1} \text{ and } F_1(y) = 1-(1-y)^{\frac{1}{\eta}}. \tag{2}$$

The functions $f_{-1}(y)$ and $f_1(y)$ represent two possible extremes for the storage bay activity profile, i.e., product stored in the rack in order of either decreasing or increasing volume. We let $f_0(y)$ denote the uniform storage bay activity profile, i.e., high volume and low volume products are evenly mixed in the rack so the expected volume by location is constant (i.e., $f_0(y) = 1$).

[2] The power distribution with $\eta = 8$ approximates the product line stratification at a firm we are familiar with. The firm has 338 SKUs. There are 25 SKUs per bay and each bay requires about 10 feet. The top 11 SKUs (3.3% of product line) make up 60% of total volume and the top 60 SKUs (17.8% of product line) make up 80% of total volume, e.g., more than 80% of the volume tends to be picked in the first 30 feet of the rack when product is stocked in order to highest-to-lowest volume. When $\eta = 8$, then $F_{-1}(y) = y^{1/\eta} = y^{1/8}$, $F_{-1}(3\%) = 64.5\%$, and $F_{-1}(17\%) = 80.1\%$.

3.3. Transformation of the Stochastic Model to a Deterministic Model

This subsection first describes the transformation of the preceding stochastic model to a deterministic model based on expected values. We then introduce a number of properties that relate the output rate to system parameters, and that characterize optimal design decisions (see the appendix for proofs and derivations).

We follow the approach of Bartholdi and Eisenstein (1996) who introduce the concepts of *worker velocity* and *position*, and use these concepts to characterize the output rate for the deterministic model. In the context of order picking, worker velocity is the number of batches picked per period and is expressed as function of position $x \in (0, 1]$ where x is the fraction of work completed on a batch by a standard worker. Bartholdi and Eisenstein (1996) show that if workers are ordered from slowest-to-fastest, then regardless of initial conditions, the system converges to a steady output rate with balanced workload (i.e., each worker repeats the same level of work content on successive batches). They also show that if worker velocity functions are constant and blocking is eliminated, then the output rate is maximized and can be expressed as the sum of individual worker velocities.

The following expressions show how expected velocity functions, which are the basis for our deterministic model, are derived from parameters and functions of the stochastic model. The functions $v(x)$, $v_1(x)$, and $v_2(x)$ are the *instantaneous* expected velocities (i.e., batches picked per period) at position x in the stochastic model.

$$\upsilon(y) = \begin{cases} \dfrac{1}{\overline{p}qf(y) + \overline{w}} \text{ for C1} \\ \dfrac{1}{\overline{p}qf(y) + \overline{w}\left[1 - F(y)^q\right]} \text{ for C2} \end{cases}$$

= instantaneous velocity of a standard worker at location y on the rack

$$G(y) = \left(\int_0^y \frac{1}{\upsilon(t)}dt\right) \Big/ \left(\int_0^1 \frac{1}{\upsilon(t)}dt\right)$$

= position as a function of location y = standard work content between location 0 and location y as a fraction of total standard work content; alternatively, $G^{-1}(x)$ = location as a function of position x

$$v(x) = \begin{cases} \dfrac{1}{\left[\bar{p}qf\left(G^{-1}(x)\right)+\bar{w}\right]G^{-1}{}'(x)} & \text{for C1} \\ \dfrac{1}{\left[\bar{p}qf\left(G^{-1}(x)\right)+\bar{w}\left(1-F\left(G^{-1}(x)\right)^q\right)\right]G^{-1}{}'(x)} & \text{for C2} \end{cases}$$

= instantaneous velocity of a standard worker at position x

$$v_1(x) = \begin{cases} \dfrac{1}{\left[(1+\rho)\bar{p}qf\left(G^{-1}(x)\right)+(1+\omega)\bar{w}\right]G^{-1}{}'(x)} & \text{for C1} \\ \dfrac{1}{\left[(1+\rho)\bar{p}qf\left(G^{-1}(x)\right)+(1+\omega)\bar{w}\left(1-F\left(G^{-1}(x)\right)^q\right)\right]G^{-1}{}'(x)} & \text{for C2} \end{cases}$$

= instantaneous velocity of a worker 1 at position x

$$v_2(x) = \begin{cases} \dfrac{1}{\left[(1-\rho)\bar{p}qf\left(G^{-1}(x)\right)+(1-\omega)\bar{w}\right]G^{-1}{}'(x)} & \text{for C1} \\ \dfrac{1}{\left[(1-\rho)\bar{p}qf\left(G^{-1}(x)\right)+(1-\omega)\bar{w}\left(1-F\left(G^{-1}(x)\right)^q\right)\right]G^{-1}{}'(x)} & \text{for C2} \end{cases}$$

= instantaneous velocity of a worker 2 at position x

The expression for $\upsilon(y)$ follows from several observations. First, $P[Y_{(q)} > y] = 1 - F(y)^q$ = probability that a worker walks to at least location y in a batch of q units given conveyor configuration C2. Thus, $\frac{1}{\bar{w}}$ and $\frac{1}{\bar{w}\left[1-F(y)^q\right]}$ are the instantaneous expected walking velocities at location y under configurations C1 and C2 respectively. Similarly, $\frac{1}{\bar{p}qf(y)}$ is the instantaneous expected picking velocity at location y. The impact of the storage bay activity profile on the velocity function $\upsilon(y)$ is reflected in the $f(y)$ and $F(y)$ terms. For the purposes of analysis, it is convenient to express velocity as a function of position. Accordingly, the functions $G(y)$ and $G^{-1\prime}(x)$ define the mapping of location y to position x, and vice-versa. The $G^{-1\prime}(x)$ term in the expressions $v(x)$, $v_1(x)$, and $v_2(x)$ above is required to properly account for the change of variable.

Let $\bar{W}$ denote the expected standard walking work content per batch, i.e.,

$$\bar{W} = \begin{cases} \bar{w} \text{ for C1} \\ \int_0^1 \bar{w}\left[1 - F(t)^q\right] dt \text{ for C2} \end{cases}. \tag{3}$$

The following property characterizes the output rate.

Property 1. *Worker 1 performs the following fraction of standard work*

$$x^* = \frac{(1-\rho)\bar{p}q + (1-\omega)\bar{W}}{2(\bar{p}q + \bar{W})}, \tag{4}$$

with worker 2 performing fraction $1 - x^*$, *and the total number of items picked per period is*

$$\pi = \frac{2}{(1-\omega^2)(\bar{p} + \bar{W}/q) - (\rho - \omega)\bar{p}\left[1 + \omega - 2F\left(G^{-1}(x^*)\right)\right]}. \tag{5}$$

We offer two observations that follow from Property 1. First, the numerator in (4) is the time it takes for worker 2 to pick a batch when working alone and the denominator is twice the time it takes for a standard worker to pick a batch. The expression implies that the ordering of workers does not affect the fraction of standard work performed by each worker. For example, for any given activity profile and conveyor configuration, a worker who does x% of standard work when in the second position will also do x% of standard work when in the first position. (However, depending on labor specialization, the activity profile, and the conveyor configuration, the ordering of workers can affect the time it takes to complete the standard work.)

Second, when there is no specialization of labor (i.e., $\rho = \omega$), the ordering of workers has no effect on the output rate, i.e., $\pi = \dfrac{2}{(1-\omega^2)(\overline{p}+\overline{W}/q)} = \dfrac{2}{(1-\rho^2)(\overline{p}+\overline{W}/q)}$.[3] In addition, under C1, the choice of activity profile has no effect on the output rate. The optimal profile for C2 is the profile that minimizes the standard walking work content $\overline{W}$; $\overline{W}$ is minimized by placing products in the rack in order of decreasing volume, i.e., the optimal profile is $f^*(y) = f_{-1}(y)$.

The optimal choices for worker ordering and storage bay activity profile are less straightforward when specialization of labor is present, especially for C2. Property 2 characterizes the optimal ordering and profile decisions and the output rate for C1.

Property 2. *For C1 with $\rho \neq \omega$, the optimal profile is $f^*(y) = f_{-1}(y)$ with workers ordered such that $\rho < \omega$. The optimal output rate is*

$$\pi = \frac{2}{\left[1+\omega-(\omega-\rho)\left(\dfrac{\left(G^{-1}(x^*)\right)^{1/\eta}}{\left(G^{-1}(x^*)\right)^{1/\eta}+G^{-1}(x^*)\overline{w}/(\overline{p}q)}\right)\right]\left[(1-\rho)\overline{p}+(1-\omega)\overline{w}/q\right]} \qquad (6)$$

Alternatively, the same output rate is achieved with profile $f^*(y) = f_1(y)$ and workers ordered such that $\rho > \omega$.

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[3] When there is no specialization of labor, then it can be shown that worker velocity functions are constant, and consequently, the analysis of Bartholdi and Eisenstein (1996) applies. They show that the output rate is the sum of worker velocities. In terms of our notation, $v_1 = 1/[(1+\omega)(\overline{p}q+\overline{W})]$, $v_2 = 1/[(1-\omega)(\overline{p}q+\overline{W})]$, and $v_1 + v_2 = 2/[(1-\omega^2)(\overline{p}q+\overline{W})]$.

To simplify interpretation, let us assume that the faster picker is not the slower walker (i.e., $\rho\omega \geq 0$), and that the difference in worker efficiency is primarily manifested in picking speed (i.e., $|\rho| > |\omega|$). This allows us allows us to unambiguously refer to a faster or slower worker. It also allows us to describe optimal decisions solely in terms of picking activity rather than both picking and walking activity. We expect that this combination of assumptions is more prevalent than the other combinations in practice. The interpretations can be extended to accommodate the other cases, but we do not take the space here.

In general, Property 2 says that output rate is maximized with a profile that accentuates the difference in relative walking and picking intensity over the rack and with workers positioned to exploit this difference. More specifically, products should be stocked in the rack either in order of lowest-to-highest volume (i.e., profile $f_1(y)$) or in order of highest-to-lowest volume (i.e., profile $f_{-1}(y)$). If profile $f_1(y)$ is implemented, then relative picking intensity is greatest at the end of the rack, and the faster picker should be in the second position (i.e., $\rho > \omega$). Alternatively, the faster picker should be in the first position if profile $f_{-1}(y)$ is implemented.

The extent to which specialization of labor can improve the output rate through judicious choice of profile and worker order is largely affected by three factors: the degree of product volume variation η, the difference in picking and walking efficiencies $|\rho - \omega|$, and the batch size q. This can be seen by comparing the output rate under a uniform profile, which maintains constant relative picking and walking intensities over the rack and consequently does not exploit specialization of labor, with the optimal output rate (see Table 1).

		High Volume Variation $\eta = 16$; e.g., $F_{-1}(.03) \approx 80\%$			Moderate Volume Variation $\eta = 8$; e.g., $F_{-1}(.17) \approx 80\%$			Low Volume Variation $\eta = 4$; e.g., $F_{-1}(.41) \approx 80\%$		
$\frac{\bar{w}}{\bar{p}}$	q	ρ=1/5	ρ=1/2	ρ=5/7	ρ=1/5	ρ=1/2	ρ=5/7	ρ=1/5	ρ=1/2	ρ=5/7
10	2	2.97%	7.41%	10.57%	2.64%	6.58%	9.37%	2.08%	5.16%	7.33%
10	4	4.94%	12.32%	17.57%	4.29%	10.68%	15.20%	3.26%	8.07%	11.45%
10	8	7.14%	17.95%	25.69%	5.95%	14.91%	21.30%	4.30%	10.67%	15.14%
10	16	8.01%	21.49%	31.47%	6.44%	16.81%	24.47%	4.46%	11.27%	16.15%
10	32	5.84%	19.25%	30.82%	5.07%	14.61%	22.34%	3.57%	9.37%	13.73%
1	2	7.59%	21.51%	32.10%	6.17%	16.54%	24.37%	4.27%	10.89%	15.70%
1	4	4.95%	17.28%	29.02%	4.42%	13.22%	20.67%	3.17%	8.43%	12.46%
1	8	2.77%	10.55%	20.52%	2.61%	8.50%	14.29%	1.96%	5.44%	8.27%
1	16	1.47%	5.72%	12.14%	1.41%	4.81%	8.54%	1.10%	3.12%	4.85%
1	32	0.76%	2.97%	6.56%	0.73%	2.55%	4.69%	0.58%	1.68%	2.64%
0.1	2	1.19%	4.65%	10.03%	1.14%	3.94%	7.09%	0.90%	2.57%	4.01%
0.1	4	0.61%	2.40%	5.32%	0.59%	2.07%	3.82%	0.47%	1.36%	2.15%
0.1	8	0.31%	1.22%	2.74%	0.30%	1.06%	1.99%	0.24%	0.70%	1.12%
0.1	16	0.16%	0.61%	1.39%	0.15%	0.54%	1.01%	0.12%	0.36%	0.57%
0.1	32	0.08%	0.31%	0.70%	0.08%	0.27%	0.51%	0.06%	0.18%	0.29%

Table 1. Percentage increase in output rate when the storage bay activity profile is changed from uniform to optimal. Both workers walk at the same speed ($\omega = 0$).

The positioning of a conveyor along the rack (C2) introduces an interaction between the storage bay activity profile and standard work content that is not present under C1. Standard picking work content is not affected by the profile, but standard walking work content is minimized by stocking in order of highest-to-lowest volume (i.e., profile $f_{-1}(y)$). The degree to which profile $f_{-1}(y)$ reduces standard walking work content relative to other profiles is affected by the degree of volume variation η and the batch size q, i.e.,

$$\bar{W} = \begin{cases} \dfrac{\bar{w}}{q/\eta + 1} \text{ for } f_{-1}(y) \\ \dfrac{\bar{w}}{q+1} \text{ for } f_0(y) \\ \dfrac{\bar{w}\eta\Gamma(q+1)\Gamma(\eta)}{\Gamma(q+\eta+1)} \approx \sqrt{2\pi}\, \dfrac{\bar{w} q^{q+\frac{1}{2}} \eta^{\eta+\frac{1}{2}}}{(q+\eta)^{q+\eta+\frac{1}{2}}} \text{ for } f_1(y) \end{cases}. \quad (7)$$

Profile $f_{-1}(y)$ minimizes work content. On the other hand, since walking intensity diminishes over the length of the rack (i.e., $\frac{1}{\overline{w}\left[1-F(y)^q\right]}$ is the instantaneous walking velocity at location y), profile $f_1(y)$ maximizes the difference in relative picking and walking intensities over the length of the rack. These competing effects inhibit a simple characterization of the optimal output rate and the choices for worker order and profile. However, the fact that profile $f_{-1}(y)$ is optimal for C2 when $\rho = \omega$, suggests that it is likely to be attractive when $\rho \neq \omega$. Property 3 gives expressions for the optimal output rate by profile.

Property 3. *For C2 with $\rho \neq \omega$, the optimal output rate by profile is*

$$f_{-1}:\ \pi = \frac{2}{(1-\omega^2)\left(\overline{p}+\overline{W}/q\right)-(\rho-\omega)\overline{p}\left(1+\omega-2\min\left\{F_{-1}\left(G^{-1}(x^*)\right),\overline{F}_{-1}\left(G^{-1}(1-x^*)\right)\right\}\right)} \tag{8}$$

$$f_0:\ \pi = \frac{2}{(1-\omega^2)\left(\overline{p}+\overline{W}/q\right)-(\rho-\omega)\overline{p}\left(1+\omega-2F_0\left(G^{-1}(x^*)\right)\right)} \tag{9}$$

$$f_1:\ \pi = \frac{2}{(1-\omega^2)\left(\overline{p}+\overline{W}/q\right)-(\rho-\omega)\overline{p}\left(1+\omega-2F_1\left(G^{-1}(x^*)\right)\right)} \tag{10}$$

where $\rho > \omega$, $\overline{F}_{-1}(y) = \int_0^y f_{-1}(1-t)\,dt$, and $\overline{W}$ as defined in (7). *The slower picker is positioned first for profiles f_0 and f_1. If $F_{-1}\left(G^{-1}(x^*)\right) = \min\left\{F_{-1}\left(G^{-1}(x^*)\right),\overline{F}_{-1}\left(G^{-1}(1-x^*)\right)\right\}$, then the slower picker is position first for profile f_{-1}; otherwise the faster picker is positioned first.*

We used Property 3 to compute the optimal output rate over a range of parameter values[4] and found that the output rate was consistently maximized with profile f_{-1} and the faster picker first. Table 2 presents a summary of insights provided by the deterministic model.

[4] $\omega = 0$, $|\rho| \in \{1/5, 1/2, 5/7\}$, $\overline{w}/\overline{p} \in \{0.1, 1, 10\}$, $q \in \{2, 4, 8, 16, 32\}$

	Conveyor at end of rack (C1)	Conveyor along rack (C2)
$\rho = \omega$	The storage bay activity profile and worker order do not affect the output rate.	Output is maximized by stocking products in order of highest-to-lowest volume; worker order does not affect the output rate.
$\rho \neq \omega$ $\rho\omega \geq 0$ $\lvert\rho\rvert > \lvert\omega\rvert$	Output is maximized by stocking products in order of highest-to-lowest volume and positioning the faster picker first. Output is also maximized by stocking products in order of lowest-to-highest volume and positioning the slower picker first.	Output is maximized by stocking products in order of highest-to-lowest volume and positioning the faster picker first.
In general, the sensitivity of output rate to the storage bay activity profile and the order of workers is increasing in labor specialization ($\lvert\rho - \omega\rvert$) and product volume variation (η), and is decreasing in the number of items picked per batch (q)		

Table 2 Summary of conclusions that follow from the deterministic model.

4. Simulation Experiments

We developed a discrete event simulation model (written in C) of a bucket brigade order picking system to gauge the robustness of the insights from the deterministic model. Our simulation model consists of 60 3-ft wide sections which represent demand points along the storage rack. These demand points are not meant to represent physical structures but, rather, the amount of space occupied by each worker when stationary (i.e., picking). The stochastic nature of our simulation model derives through sampling from a discrete version of the power distribution described in Subsection 3.2 to form the batches, (i.e., in contrast to the deterministic model, demand points are discretely distributed along the storage rack). We assume the standard bucket brigade protocol of no passing and preemption. That is, if worker 1 arrives at a section of storage rack occupied by worker 2, then worker 1 stands idle until worker 2 completes the picking requirements in that section. Further, if a handoff occurs while worker 1 is engaged in picking, the remaining time to pick the item(s) is determined using the picking rate of worker 2 and worker 1 immediately begins to walk back to the beginning of the rack to retrieve the next batch. When workers walk at different speeds, occasionally a worker will catch another while walking. In these instances we take a conservative approach and assume that the faster walker inherits the speed of the slower walker for the duration of this condition. Finally, we assume that the standard pick time is a constant 10 sec/item and the standard walking speed, which is constant regardless of location or direction, is adjusted to obtain the values of $\overline{w} / \overline{p}$ dictated by the experiment design described below.

The simulation is initialized in an empty and idle state. Steady state is reached quickly (typically within the first 20 – 30 batches). The simulation is very efficient, taking approximately 1 second to process 1,000 batches with a computer running at1.8 GHz. Therefore, to be conservative, we fixed the initialization period at 2,000 orders. After the completion of the initialization period, all performance variables are cleared and the next 5,000 orders are used to generate the data used in our analysis.

We computed output rates using the deterministic model and the simulation model over the following 512 combinations of values: $\overline{w} / \overline{p} \in \{1, 10\}$, $q \in \{4, 16\}$, $\omega \in \{0, \rho\}$, $|\rho| \in \{1/5, 1/2\}$, $\eta \in \{1, 4, 8, 16\}$, C$i \in$ {C1, C2}, $f_i(y) \in \{f_{-1}(y), f_1(y)\}$, position of slower worker ∈ {1$^{\text{st}}$, 2$^{\text{nd}}$}. Note that the output rates at $\eta = 1$ are also the output rates for the uniform profile $f_0(y)$. Our experiment design is constructed such that the range of parameter values is wide enough to discern meaningful effects while being consistent with our interpretation of industry norms. The rack length (i.e., $\overline{w} / \overline{p}$) is a possible exception. We expect that many racks used in industry would be longer than our "long" rack (i.e., the time to walk the length of the rack and back is typically more than 10 times the time to pick a single item). However, we are interested in changes in output as conditions become more congested. We find that for $\overline{w} / \overline{p} = 10$, blocking is largely eliminated under a variety of parameter combinations making $\overline{w}/\overline{p}$ = 10 a reasonable, though perhaps conservative, baseline for systems with low congestion.

The following four figures show the results of our experiment. Each figure contains four graphs displaying the maximum output rate among the choices for worker ordering and storage bay activity profile as a function of η and conveyor location. The figures correspond to (1) short rack / small batch size ($\overline{w}/\overline{p}$=1, q = 4), (2) short rack / large batch size ($\overline{w}/\overline{p}$=1, q = 16), (3) long rack / small batch size ($\overline{w}/\overline{p}$=10, q = 4), (4) long rack / large batch size ($\overline{w}/\overline{p}$=10, q = 16). Figures 3 and 4 illustrate system performance when there is no specialization of labor (i.e., $|\rho| \in \{1/5, 1/2\}$ and $\omega = \rho$). Figures 5 and 6 illustrate system performance when specialization of labor can play a role (i.e., $|\rho| \in \{1/5, 1/2\}$ and $\omega = 0$).

In the following subsections, we interpret the results with respect to the accuracy of the deterministic model and the choices of worker ordering and storage bay activity profile for each conveyor configuration. The implications of changes in the batch size are discussed as well.

— figures 3 – 6 about here —

4.1. Deterministic Model versus Simulation Model Results

When blocking is largely eliminated, the match between the output rates predicted by the deterministic model and those observed from the simulation model is nearly exact, and the output rates increasingly diverge as blocking increases. This observation follows from examination of the detailed results, but evidence of the phenomenon can also be seen in the close alignment of the deterministic and simulation output rates in many of the figures. This is a meaningful result. The deterministic model is a continuous and deterministic analog of a stochastic system. It assumes that each worker repeats the same interval of work on each successive batch with work times and pick locations set according to expected values. In real warehouses, and in our simulation model, this does not occur (we examined a number of handoff point distributions and found no evidence of convergence to a fixed point). However, when design choices are made such that blocking is negligible, workers are nearly constantly working and the system behaves substantially as if it did.

4.2. Choice of Worker Ordering and the Storage Bay Activity Profile

The choice of worker ordering and storage bay activity profile is influenced by many factors, and blocking can play a major role. We focus first on the case where the conveyor is located at the end of the rack (C1), and then we consider the case where a take-away conveyor is added (C2). For each case, we begin by summarizing the predictions from the deterministic model regarding optimal choices for worker ordering and storage bay activity profile, and then we contrast these predictions with the results from the simulation.

Conveyor Configuration 1 (C1)

The deterministic model predicts that neither the worker ordering nor the storage bay activity profile affects the output rate when there is no specialization of labor ($\rho = \omega$); as specialization of labor is introduced ($\rho \neq \omega$), the deterministic model predicts that output is maximized by ordering workers such that the fastest picker's work interval contains the highest volume items in the storage rack. When the profile is uniform, there is no opportunity to exploit specialization of labor.

In the absence of labor specialization worker ordering and profile choices for a bucket brigade should be made on the basis of a single criterion—minimize blocking. The results in figures 3 and 4 show that simulation output is consistently maximized (and blocking is minimized) when workers are ordered from slowest-to-fastest and when the storage bay activity profile is uniform.

The preference for a slowest-to-fastest worker ordering is not surprising; positioning the slowest worker first reduces the likelihood that the first (slowest) worker catches and is blocked by the second (fastest) worker. The impact of the storage bay activity profile on simulation output is more subtle. We see that curves for the C1 simulation output rates are generally flat over η, indicating that this parameter has little impact of output. An exception to this, which is especially noticeable in Figure 3, occurs when the rack is short (i.e., $\overline{w} / \overline{p} = 1$) and product volume variation is high (i.e., $\eta = 8$ or 16). In these instances we see that the C1 simulation output curve decreases as the profile shifts from uniform profile $f_0(y)$ to the skewed profile $f_1(y)$.[5] The uniform profile spaces out the picking activity over the rack, thus reducing congestion and blocking, and resulting in a higher output rate.

The introduction of labor specialization should have little effect on the preference for slowest-to-fastest worker ordering; the deterministic model predicts that worker ordering does not affect the optimal output rate given the appropriate storage bay activity profile, and slowest-to-fastest worker ordering is effective for reducing blocking. This leaves two potentially conflicting criteria that influence the choice of storage bay activity profile. On one hand, the uniform profile $f_0(y)$ is effective for reducing blocking. One the other hand, the skewed profile $f_1(y)$ is effective for exploiting specialization of labor. From Property 2, we expect that it is more likely for $f_1(y)$ to be preferred over $f_0(y)$ when $\overline{w} / \overline{p}$, η, and $|\rho - \omega|$ are high. Indeed, this is borne out in Figures 5 and 6. We see that simulation output with profile $f_0(y)$ is as high as or higher than with profile $f_1(y)$ in Figure 5 where $|\rho - \omega| = 1/5$, and simulation output with profile $f_1(y)$ is generally as high as or higher than with profile $f_0(y)$ in Figure 6 where $|\rho - \omega| = 1/2$. However,

[5] The difference in simulation output rates between $f_0(y)$ and $f_1(y)$ is noticeably smaller when $|\rho| = 1/2$ than when $|\rho| = 1/5$. This can be explained by the fact that as the difference in worker speeds increases, it is less likely for the slowest worker, who is in the first position, to be blocked by the fastest worker.

Figure 6 also shows that $f_1(y)$ becomes increasingly attractive relative to $f_0(y)$ as $\overline{w} / \overline{p}$ and η increase. Profile $f_1(y)$ exploits labor specialization and avoids blocking by concentrating work toward the end of the rack keeping the slower worker occupied in the portion of the rack where there is less picking intensity.

In summary, slowest-to-fastest worker ordering coupled with a uniform storage bay activity profile are the best choices for a bucket brigade under many conditions when a take-away conveyor is not present. This combination of design choices consistently eliminates blocking and produces output rates that nearly match the deterministic model. An exception to the superiority of a uniform profile can occur when labor specialization is present. Profile $f_1(y)$ can increase output by exploiting specialization of labor and is less susceptible to blocking than profile $f_{-1}(y)$.

Conveyor Configuration 2 (C2)

When a take-away conveyor is introduced, the storage bay activity profile affects the amount of walking required to retrieve each batch. Expected walking work content is minimized with profile $f_{-1}(y)$, and the deterministic model suggests that the sensitivity of the output rate to changes in the profile is decreasing in q, and increasing in product volume variation and rack length.

The deterministic model predicts that worker ordering does not affect the output rate when there is no specialization of labor and, as specialization of labor is introduced, workers should be ordered so that the fastest picker does relatively more picking and less walking. Thus, with no specialization of labor, the optimal design choices according to the deterministic model are profile $f_{-1}(y)$ and any worker order.

In general, the choice of the storage bay activity profile introduces a trade-off between blocking and walking work content when a take-away conveyor is used in conjunction with a bucket brigade. As the walking content is reduced, congestion and, consequently, the incidence of blocking increase and the effectiveness of the bucket brigade deteriorates.

Figures 3 and 4 show that the preference for $f_{-1}(y)$ persists in the simulation output due to the reduction in walking effort it engenders. However, slowest-to-fastest worker ordering is generally preferred for reducing blocking. Further, we see that in many cases (even with slowest-to-fastest worker ordering) a high incidence of blocking remains and the bucket brigade is unable to attain the potential increases in output that this conveyor configuration presents. Nonetheless, when the rack is long, the introduction of a take-away conveyor can be expected to reduce walking effort and increase productivity substantially in spite of the increase in congestion and blocking it causes (however particularly large batches, i.e., $q = 16$, tend to increase the range of bays visited and, therefore, the walking required, offsetting much of the effect). Figures 3 and 4 show that when the rack is short ($\overline{w} / \overline{p} = 1$), congestion effects dominate and a uniform profile results in the highest output.

We find that the preceding conclusions are relatively unaffected by the introduction of labor specialization (except that the gap between potential and realized production rates increases in many cases). Profile $f_{-1}(y)$ is necessary to reduce the walking content, and specialization of labor can only be exploited by positioning the fastest worker first. In particular, figures 5 and 6 show that simulation output is generally highest when the slowest worker is first. As in the case where $\rho = \omega$, the output is sometimes higher when the faster worker is first (e.g., when the rack is long, the batch size is small). In combination with profile $f_{-1}(y)$, this ordering of workers matches the fastest picker with the section of the rack that has the highest picking intensity, thus more effectively exploiting labor specialization. Also, as in the case where $\rho = \omega$, the uniform profile results in the highest output rate when the rack is short ($\overline{w}/\overline{p}$ = 1) and the profile $f_{-1}(y)$ generally results in the highest output when the rack is long ($\overline{w}/\overline{p}$ = 10).

Interestingly, when $\overline{w}/\overline{p}$ = 10 and q = 16, output rates are a little higher with profile $f_1(y)$ than with profile $f_{-1}(y)$. This is because changes in the profile have little effect on the expected walking work content when the batch size is large, and because profile $f_1(y)$ reduces blocking affording greater opportunity to exploit labor specialization.

In summary, we see some similarities and some notable differences in the preferred design choices for C2 compared to the case of C1. Under C1, the design guidelines for bucket brigades are straightforward: arrange the workers from slowest-to-fastest along the rack, and unless there are large differences in worker picking speeds, use a uniform storage bay activity profile. In cases where differences in worker picking speeds are large and the rack is long, store products in increasing order of volume.

When a take-away conveyor is introduced (C2), we find similarities to the conclusions drawn under C1 in that the uniform profile and slowest-to-fastest ordering of workers is preferred, at least when the rack is short. This combination of design choices is effective for mitigating blocking, and blocking can be a significant impediment to the output rate when the rack is short. We find differences in that a skewed profile is more likely to improve the output rate (relative to uniform), particularly a profile that concentrates high movers near the beginning of the rack and slow movers near the end of the rack. In these settings, we also find that output can sometimes be improved by positioning the fastest worker first, i.e., the benefits of improved labor specialization with fastest-to-slowest worker ordering more than offset the drawbacks of increased blocking.

Finally, as noted at the beginning of this paper, one guiding principle from the literature is to increase the batch size to the point where either the number of items in a batch becomes difficult to move or the batches in process approach available space constraints. This fact can be clearly seen in figures 3-6 where the output associated with $q = 16$ is consistently higher than when $q = 4$.

5. Summary and Conclusions

In this paper we develop a deterministic model for predicting the output rate of an order picking system in an e-commerce fulfillment center. Given the significant amount of walking encountered in these systems, we separate work into two components—walking and picking—and analyze expressions for the output rate under a variety of environmental conditions and managerial decisions covering the bulk of options available for improving warehouse performance. We then analyze the robustness of the predictions of the deterministic model with a discrete event simulation model designed to mimic their operation in real fulfillment centers. The results lend insights into the management of bucket brigades. In particular, we find that the deterministic model can be a valuable tool for understanding and improving system performance.

When the standard conveyor configuration is in place (i.e., C1), we find that the output rate is often maximized with a uniform storage bay activity profile and a slowest-to-fastest ordering of workers. In cases where the rack is long and differences between picking and walking speeds are significant, it is possible to increase the output rate by concentrating high movers near the end of the rack and slow movers near the beginning of the rack. In general, we find that the optimal choices for worker ordering and storage bay activity profile largely eliminate blocking, and as a result, the simulation output rate is accurately predicted by the deterministic model (i.e., the bucket brigade performs optimally).

Guidelines for worker ordering and storage bay activity profile become more complicated with the introduction of a take-away conveyor. When the rack is short, manipulating the storage bay activity profile and adding a take-away conveyor present less potential to improve the output rate. Further, congestion and blocking are more likely to occur. As a result, the output of bucket brigades is generally maximized with a uniform storage bay activity profile and a slowest-to-fastest ordering of workers and the introduction of a take-away conveyor has minimal impact.

When the rack is long, adding a take-away conveyor can improve the output rate considerably and it may be possible to increase the output rate by concentrating high movers near the beginning of the rack and slow movers near the end of the rack. Workers are still positioned from slowest-to-fastest in the bucket brigade unless there are large differences in worker picking and walking speeds; in these instances, specialization of labor benefits can outweigh the drawback of increased blocking, resulting in higher output when workers are positioned from fastest-to-slowest.

6. Appendix

Proof of Property 1. We require the following additional notation:

$\tau_i(x, x')$ = work time of worker i as function of position interval $(x, x']$ = $\int_x^{x'} \frac{1}{v_i(t)} dt$

$\tau(x, x')$ = standard work time as function of position interval $(x, x']$ = $\int_x^{x'} \frac{1}{v(t)} dt$

$\theta(x, x')$ = average percentage increase in worker 1 time relative to the standard over position interval $(x, x']$ = $[\tau_1(x, x') - (x'-x)\tau]/[(x'-x)\tau]$ = average percentage decrease in worker 2 time relative to the standard over position interval $(x, x']$

The value of x^* satisfying $\tau_1(0, x^*) = \tau_2(x^*, 1)$ is unique because $\tau_1(0, 0) = \tau_2(1, 1) = 0$, $\tau_1(0, x)$ is continuous and increasing in $x \in (0, 1]$, and $\tau_2(x, 1)$ is continuous and decreasing in $x \in (0, 1]$. Furthermore,

$$\tau_1(0, x) = \int_0^x \frac{1}{v_1(t)} dt = x\tau[1 + \theta(0, x)]$$

$$\tau_2(x, 1) = \int_x^1 \frac{1}{v_2(t)} dt = \tau_2(0, 1) - \tau_2(0, x) = \tau[1 - \theta(0, 1)] - x\tau[1 - \theta(0, x)].$$

Setting $\tau_1(0, x) = \tau_2(x, 1)$ and solving for x yields

$$x^* = [1 - \theta(0,1)]/2 = \frac{\tau_2(0,1)}{2\tau} = \frac{(1-\rho)\bar{p}q + (1-\omega)\bar{W}}{2(\bar{p}q + \bar{W})}.$$

Rearranging the above, we get

$$x^*\tau = \tau_2(0,1)/2,$$

and the cycle time can be written as

$$\tau^* = \tau_1(0, x^*) = x^*\tau[1 + \theta(0, x^*)] = 0.5\left\{(1-\omega^2)(\bar{p}q + \bar{W}) - (\rho - \omega)\bar{p}q\left[1 + \omega - 2F\left(G^{-1}(x^*)\right)\right]\right\}.$$

Thus, the output rate is

$$\pi = q/\tau^* = \frac{2}{(1-\omega^2)(\bar{p} + \bar{W}/q) - (\rho - \omega)\bar{p}\left[1 + \omega - 2F\left(G^{-1}(x^*)\right)\right]}. \ \square$$

Proof of Property 2. Since $\rho \neq \omega$, we assume without loss of generality that $\rho < \omega$ (i.e., the optimal cycle time and output rate are the same if $\rho > \omega$, see Property 6 in Webster, Ruben, and Yang 2006). If $\rho < \omega$ (e.g., the faster picker and/or slower walker is first), then the optimal profile stores product in order of highest-to-lowest volume (see Property 7 in Webster, Ruben, and Yang 2006). Substituting the optimal profile $F_{-1}(y) = y^{\frac{1}{\eta}}$ into (5) gives (6). ❑

Proof of Property 3. Expressions (8) – (10) follow from appropriate substitution into (5) as governed by the following facts. For profile $f_1(y)$ (i.e., products arranged in order of lowest-to-highest volume), the optimal ordering of workers is such that $\rho > \omega$ (i.e., the slower picker and/or faster walker is first; see Webster, Ruben, and Yang 2006). For profile $f_0(y)$ (i.e., product volumes are uniform over the rack), the optimal ordering of workers is such that $\rho > \omega$ (i.e., the slower picker and/or faster walker is first; see Webster, Ruben, and Yang 2006). (The intuition underlying the preceding two results is that, for both profiles, picking intensity is greatest and walking intensity is lowest near the end of the rack, and thus the faster picker and/or slower worker should be positioned last.) For profile $f_{-1}(y)$ (i.e., products arranged in order of highest-to-lowest volume), the optimal ordering of workers is parameter dependent. The min operator in (8) accounts for the optimal ordering of workers. ❑

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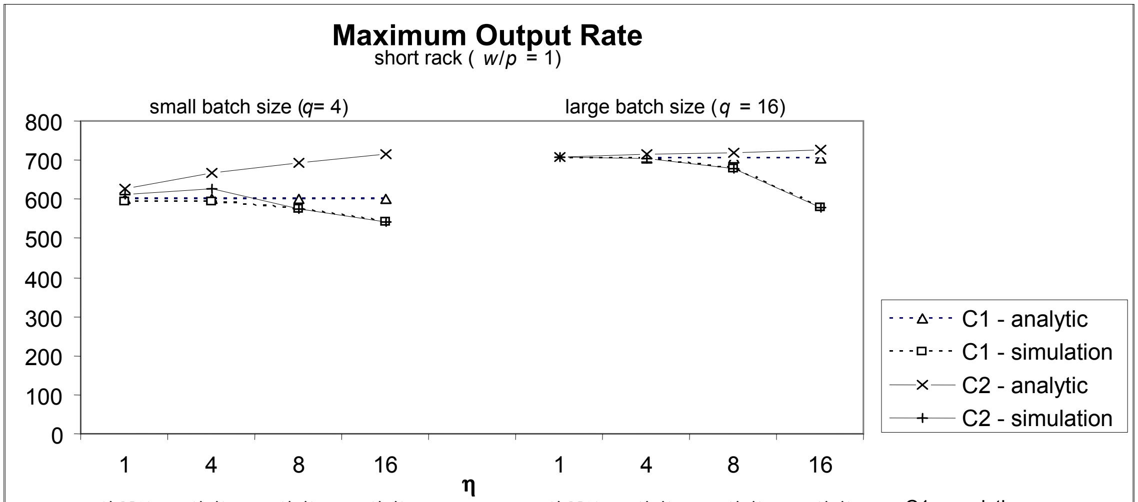
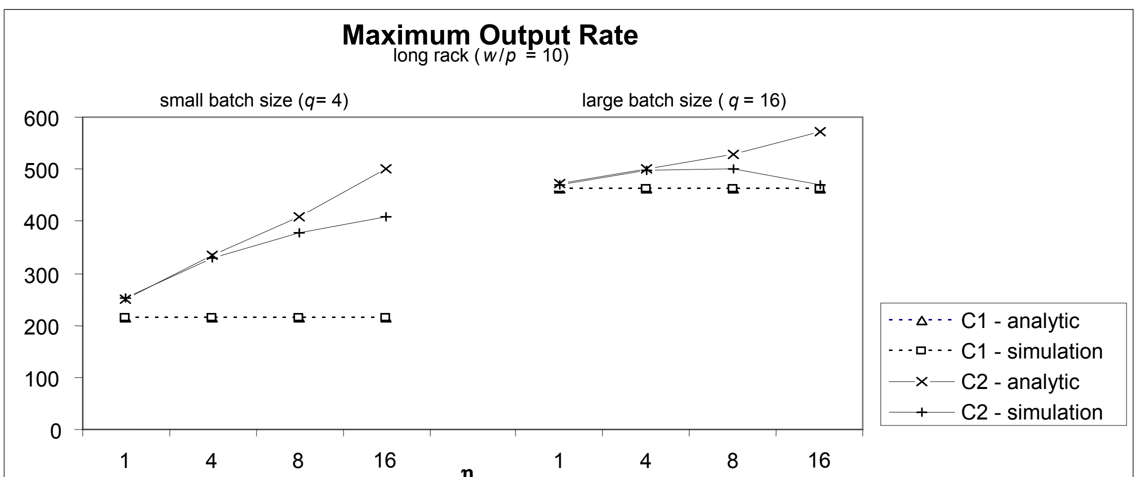
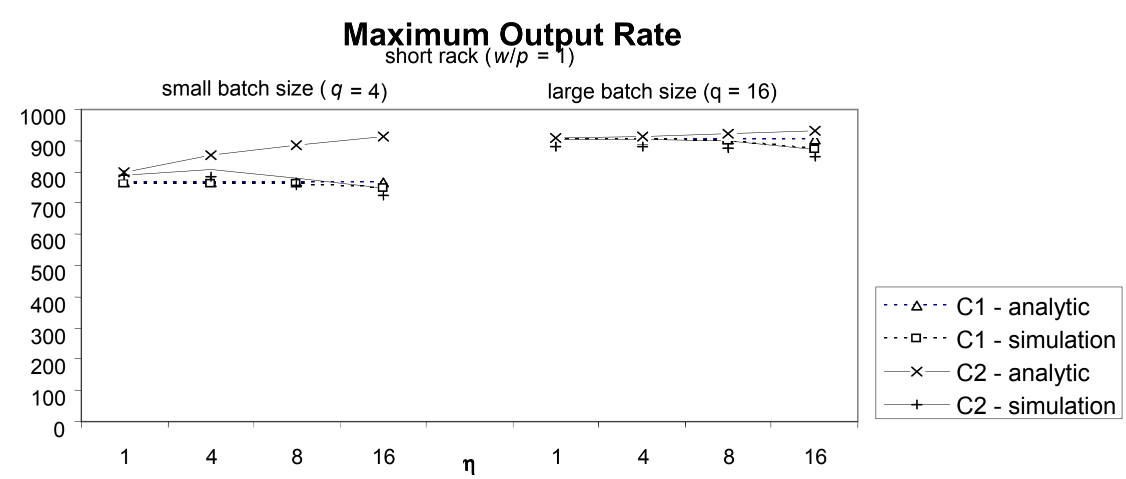


η = 1	4	8	16	1	4	8	16	
(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	C1 - analytic
(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C1 - simulation
(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	C2 - analytic
(↑,NA)	(↑,↓)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C2 - simulation

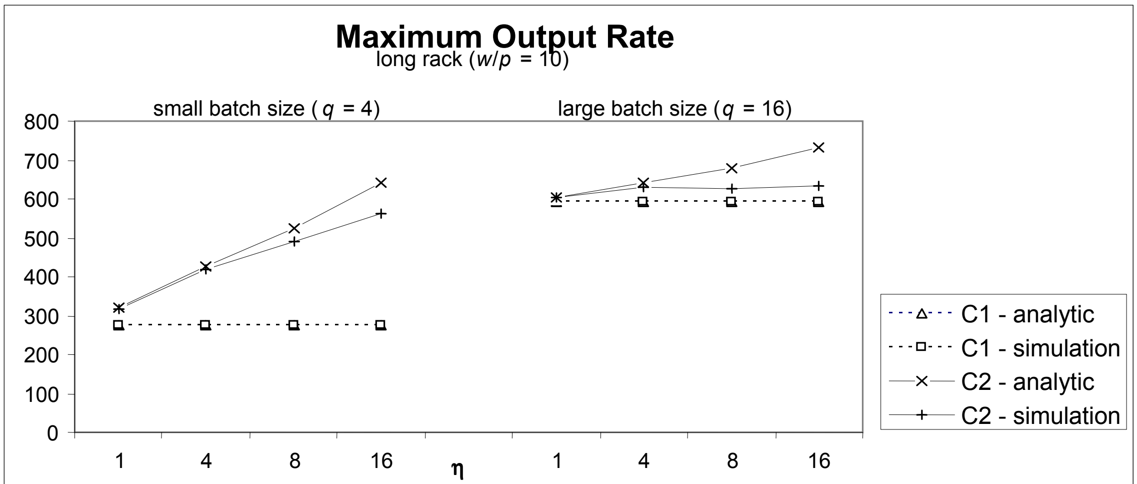


η = 1	4	8	16	1	4	8	16	
(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	C1 - analytic
(↑,NA)	(↑,↕)	(↑,↕)	(↑,↕)	(↑,NA)	(↑,↕)	(↑,↑)	(↑,↑)	C1 - simulation
(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	C2 - analytic
(↓,NA)	(↓,↓)	(↓,↓)	(↑,↓)	(↑,NA)	(↑,↓)	(↑,↓)	(↑,↓)	C2 - simulation

Figure 3. Output rates for the deterministic model and the simulation by rack length, conveyor location, batch size, and volume variation in the product line when $|\rho| = 1/5$ and $\omega = \rho$. The output rates displayed are the maximum among the 4 choices of worker order and storage bay activity profile, and the optimal worker order and profile decisions are identified in the table below each chart, e.g., (↑,↓) = workers ordered from slowest-to-fastest and products arranged from high movers to slow movers and (↕,↕) = worker ordering and storage bay activity profile do not affect output.

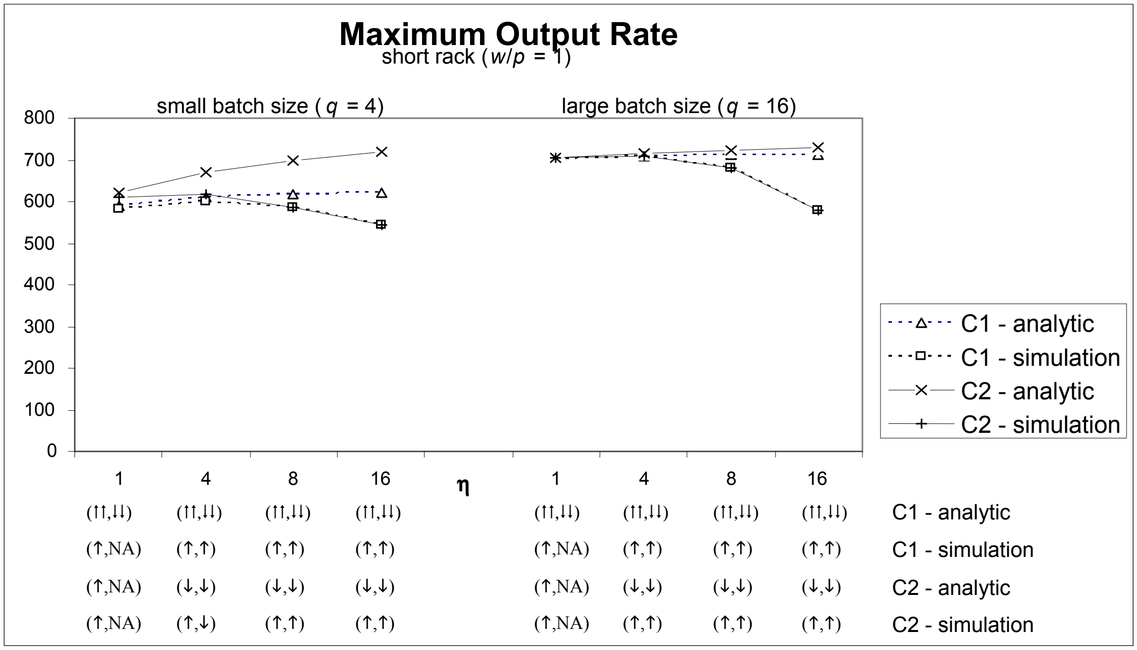


(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	C1 - analytic
(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C1 - simulation
(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	C2 - analytic
(↑,NA)	(↑,↓)	(↑,↓)	(↑,↓)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C2 - simulation



(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	(↕,NA)	(↕,↕)	(↕,↕)	(↕,↕)	C1 - analytic
(↑,NA)	(↑,↕)	(↑,↕)	(↑,↕)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C1 - simulation
(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	(↕,NA)	(↕,↓)	(↕,↓)	(↕,↓)	C2 - analytic
(↑,NA)	(↑,↓)	(↑,↓)	(↑,↓)	(↑,NA)	(↑,↓)	(↑,↓)	(↑,↓)	C2 - simulation

Figure 4. Output rates for the deterministic model and the simulation by rack length, conveyor location, batch size, and volume variation in the product line when $|\rho| = 1/2$ and $\omega = \rho$. The output rates displayed are the maximum among the 4 choices of worker order and storage bay activity profile, and the optimal worker order and profile decisions are identified in the table below each chart, e.g., (↑,↓) = workers ordered from slowest-to-fastest and products arranged from high movers to slow movers and (↕,↕) = worker ordering and storage bay activity profile do not affect output.



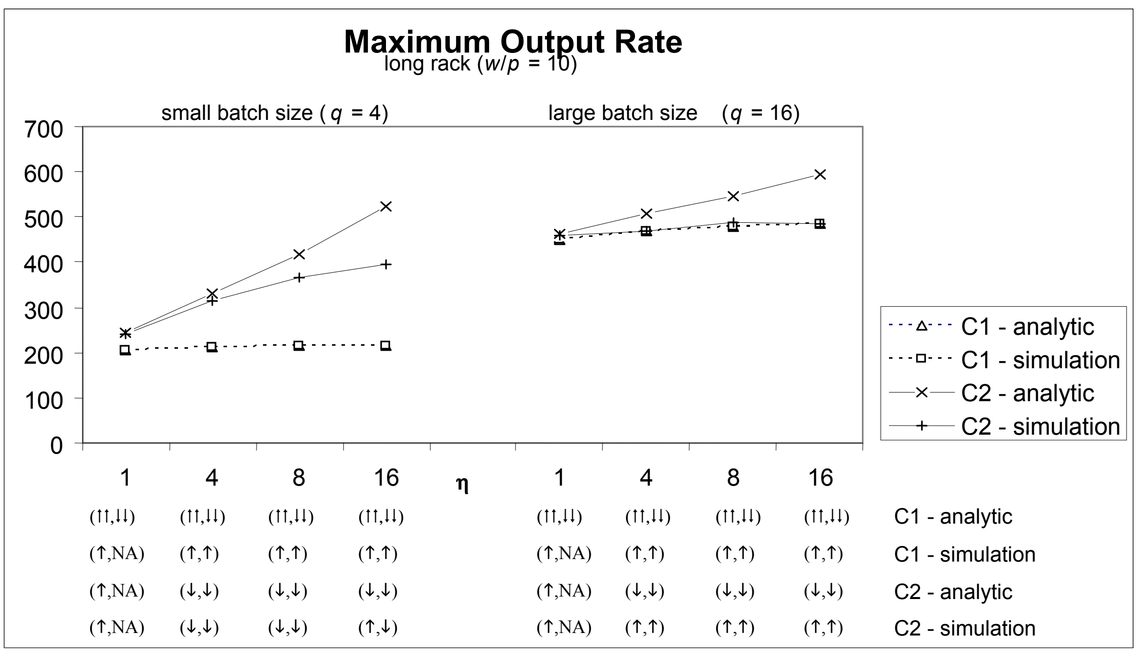
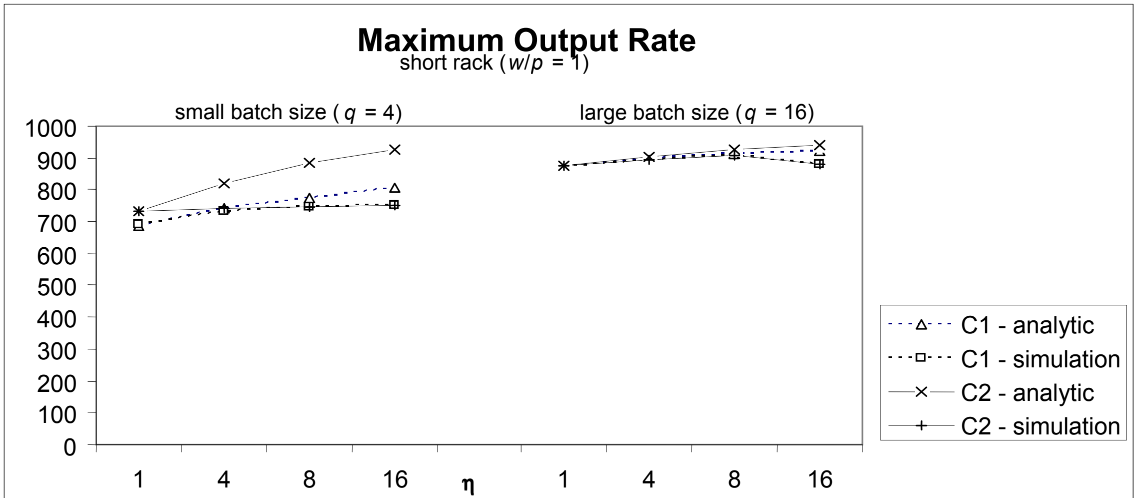
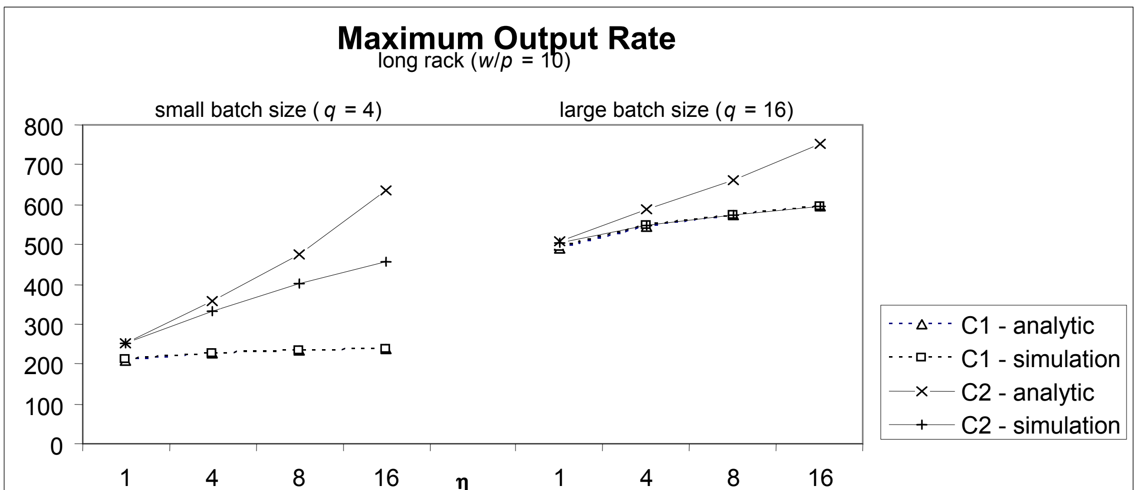


Figure 5. Output rates for the deterministic model and the simulation by rack length, conveyor location, batch size, and volume variation in the product line when $|\rho| = 1/5$ and $\omega = 0$. The output rates displayed are the maximum among the 4 choices of worker order and storage bay activity profile, and the optimal worker order and profile decisions are identified in the table below each chart, e.g., (↑,↓) = workers ordered from slowest-to-fastest and products arranged from high movers to slow movers and (↑↑,↓↓) = slowest-to-fastest worker ordering coupled with an increasing storage bay activity profile yields the same output as its mirror image.



(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	C1 - analytic
(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C1 - simulation
(↑,NA)	(↓,↓)	(↓,↓)	(↓,↓)	(↑,NA)	(↓,↓)	(↓,↓)	(↓,↓)	C2 - analytic
(↑,NA)	(↑,↓)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C2 - simulation



(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	(↑↑,↓↓)	C1 - analytic
(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C1 - simulation
(↑,NA)	(↓,↓)	(↓,↓)	(↓,↓)	(↑,NA)	(↓,↓)	(↓,↓)	(↓,↓)	C2 - analytic
(↑,NA)	(↓,↓)	(↓,↓)	(↑,↓)	(↑,NA)	(↑,↑)	(↑,↑)	(↑,↑)	C2 - simulation

Figure 6. Output rates for the deterministic model and the simulation by rack length, conveyor location, batch size, and volume variation in the product line when $|\rho| = 1/2$ and $\omega = 0$. The output rates displayed are the maximum among the 4 choices of worker order and storage bay activity profile, and the optimal worker profile decisions are identified in the table below each chart, e.g., (↑,↓) = workers ordered from slowest-to-fastest and products arranged from high movers to slow movers and (↑↑,↓↓) = slowest-to-fastest worker ordering coupled with an increasing storage bay activity profile yields the same output as its mirror image.